Formale Ableitungen

Bestimme für die gegebenen Funktionen die angegebenen Ableitungen:

1. $f_t(x) = txe^{-tx+1}$ ges. f_t', f_t'', f_t'''
2. $f_a(x) = a \cdot \ln(x^2 + a) - a$ ges. f_t', f_t'', f_t'''
3. $f_t(x) = \frac{10 - t^2x^2}{x^3}$ ges. f_t', f_t'', f_t'''
4. $f_k(x) = \frac{1}{k}x - x \cdot \ln x$ ges. f_t', f_t'', f_t'''
5. $f_t(x) = (tx - 1) \cdot e^{tx+1}$ ges. f_t', f_t'', f_t'''
6. $f_a(x) = a \cdot \sqrt{x} - \ln x$ ges. f_t', f_t'', f_t'''
7. $f_t(x) = (5 - t)\sqrt{t - x}$ ges. f_t', f_t'', f_t'''
   Diskutiere, ob das Produkt $f_t'(x) \cdot f_t(x)$ mit $x < t$ positive Werte annehmen kann.
8. $f_a(x) = \frac{10x}{(x + a)^2}$ ges. f_t', f_t''
9. $f_a(x) = \frac{2 \cdot e^{at}}{e^{at} + 29}$ ges. f_t', f_t''
10. $f_t(x) = \frac{e^x}{8 \cdot (t + x)^2}$ ges. f_t', f_t''

Lösungen:

$f_t'(x) = t \cdot (1 - tx) \cdot e^{-tx+1}$ $f_t''(x) = -t^2(2 - tx)e^{-tx+1}$ $f_t'''(x) = t^3(3 - tx)e^{-tx+1}$

$f_a'(x) = \frac{2ax}{x^2 + a}$ $f_a''(x) = \frac{2a(a - x^2)}{(x^2 + a)^2}$ $f_a'''(x) = \frac{4ax^3 - 12a^2x}{(x^2 + a)^3}$

$f_t'(x) = \frac{t^2x^2 - 30}{x^4}$ $f_t''(x) = \frac{120 - 2t^2x^2}{x^5}$ $f_t'''(x) = \frac{6t^2x^2 - 600}{x^6}$

$f_k'(x) = \frac{1}{k} - 1 - \ln x$ $f_k''(x) = -\frac{1}{x}$ $f_k'''(x) = \frac{1}{x^2}$

$f_t'(x) = t^2xe^{tx+1}$ $f_t''(x) = (t^3x + t^2) \cdot e^{tx+1}$ $f_t'''(x) = (t^4x + 2t^3) \cdot e^{tx+1}$

$f_a'(x) = \frac{a}{2\sqrt{x}} - \frac{1}{x}$ $f_a''(x) = -\frac{a}{4x\sqrt{x}} + \frac{1}{x^2}$ $f_a'''(x) = \frac{3a}{8x^2\sqrt{x}} - \frac{2}{x^3}$

$f_t'(x) = -\frac{5 - t}{\sqrt{t - x}}$ $f_t''(x) = \frac{5 - t}{2} \cdot \frac{1}{(t - x)\sqrt{t - x}}$ $f_t'''(x) = -\frac{15 - 3t}{4} \cdot \frac{1}{(t - x)^2\sqrt{t - x}}$

das Produkt ist stets negativ

$f_a'(x) = \frac{10(a - x)}{(x + a)^3}$ $f_a''(x) = \frac{20 \cdot (x - 2a)}{(x + a)^4}$

$f_a'(x) = \frac{58ae^{at}}{(e^{at} + 29)^2}$ $f_a''(x) = \frac{58a^2e^{at} \cdot (29 - e^{at})}{(e^{at} + 29)^3}$

$f_t'(x) = \frac{e^x(t + x - 2)}{8 \cdot (t + x)^3}$ $f_t''(x) = \frac{e^x \cdot (t^2 + 2tx + x^2 - 4t - 4x + 6)}{8 \cdot (t + x)^4}$